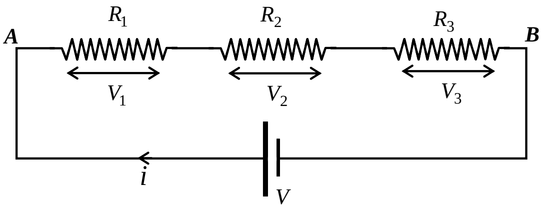
3. Grouping of Resistors & Kirchhoff's Rule

1. Grouping of resistances:

(a) Series Grouping: Supposed the resistances R_1, R_2 and R_3 are grouped in series and a pd V is applied between A and B. Current through all the resistors will be the same. Let it be i.
The potential drop across different resistors are given by: $V_1 = i_1 R_1$, $V_2 = i_2 R_2$ and $V_3 = i_3 R_3$.
Therefore net potential drop across the combination (between A and B) will be:

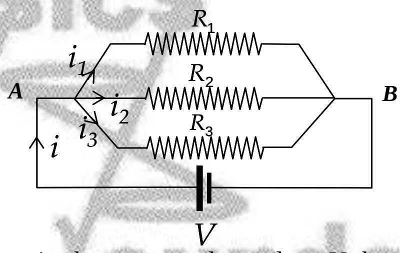
$$V = V_1 + V_2 + V_3 = i(R_1 + R_2 + R_3) \Rightarrow \frac{V}{i} = R_1 + R_2 + R_3 .$$



Since V/i = equivalent resistance between A and B. $\therefore \Rightarrow R_{eq} = R_1 + R_2 + R_3$

For n resistors in series, $R_{eq} = \sum_{i=1}^{n} R_i$

(b) Parallel Grouping: Let there be resistances R_1, R_2, R_3 grouped in parallel between A and B and pd applied across the combination is V.



Therefore, pd across each resistor is the same and equal to V, hence current through them will be different. If total current flowing through the grouping is i and supposed i_1, i_2, i_3 are the currents through R_1, R_2, R_3 respectively, then,

$$i = i_1 + i_2 + i_3 . \Rightarrow i = V/R_1 + V/R_2 + V/R_3 \quad \Rightarrow \quad i/V = 1/R_1 + 1/R_2 + 1/R_3$$

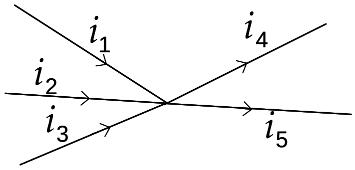
Since $\frac{i}{V} = \frac{1}{R_{eq}}$, therefore, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$.

For n resistors in parallel, $\frac{1}{R_{eq}} = \sum_{i-1}^{n} \frac{1}{R_{eq}}$.

2. Kirchhoff's Rule.

(a) Point- rule: *In a circuit, algebraic sum of total currents meeting at a point is zero*.
Or, the *sum of incoming currents = sum of outgoing currents.*
This is based on *charge conservation principle*.
Explanation: According to charge conservation principle, no point in a circuit with steady current can be a *source* or a *sink* of charges. Therefore, total charge arriving at a point per second must be equal to those leaving the point.
In other words, net current arriving at a point is equal to the net current leaving that point.
For example, if currents i_1, i_2, i_3, arrive at a point and i_4, i_5 leave that point, then,

$$i_1 + i_2 + i_3 = i_4 + i_5. \text{ Or, } i_1 + i_2 + i_3 - i_4 - i_5 = 0.$$



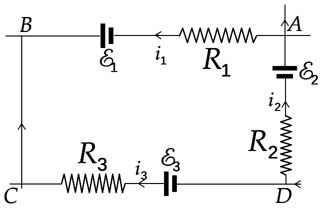
Taking incoming currents to be +*ve*, and outgoing currents to be −*ve*, the point rule can be generalised as : $\sum i = 0$.

(b) Mesh or loop rule: In a closed circuit, sum of the *potential drops* (current × resistance) across the resistors is equal to the net *emf* inserted in the loop.

$$\Rightarrow \sum iR = \sum E .$$

This rule is based on the *energy conservation principle*.

Explanation: - Potential drop (*iR*) represents total energy dissipated per unit charge flow in a resistor. Therefore the total potential drop represents total energy dissipated per unit charge in the circuit. By energy conservation principle, this must be equal to the energy supplied by the *emf seats* of the circuit per unit charge flow, i.e., the **net emf** of the circuit.



In the given mesh, taking the direction of current sent by E_1 to be +ve, the net emf of the circuit is $E_1 + E_3 - E_2$, sending current anticlockwise.

Total potential drop while traversing the loop anticlockwise is $i_1R_1 - i_3R_3 + i_2R_2$.

Hence by loop rule $i_1R_1 - i_3R_3 + i_2R_2 = E_1 - E_3 + E_2$.

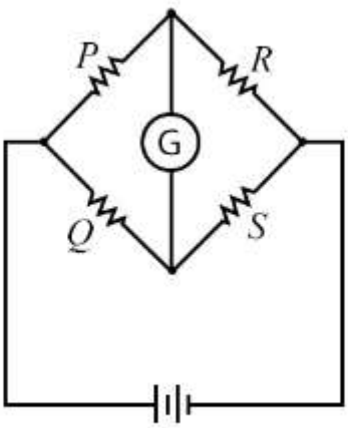
In general, $\sum iR = \sum E$.

Note: The Kirchhoff's loop rule can be stated alternatively as follows: ***Sum of the potential drops across the resistors and emf seats in a closed loop is always zero***.
Starting from point *A* we have total potential drops, $-i_1R_1 + E_1 - E_3 + i_3R_3 - i_2R_2 + E_2 = 0$.
Or, $i_1R_1 - i_3R_3 + i_2R_2 = E_1 - E_3 + E_2$.

3. Wheatstone's bridge:

A four-resistance mesh used to measure unknown resistance is called a 'Wheatstone's bridge'.

Construction: It consists of four arms forming a rectangle, each containing a resistance. One pair of diagonal points is connected to emf terminals and the other to a galvanometer.

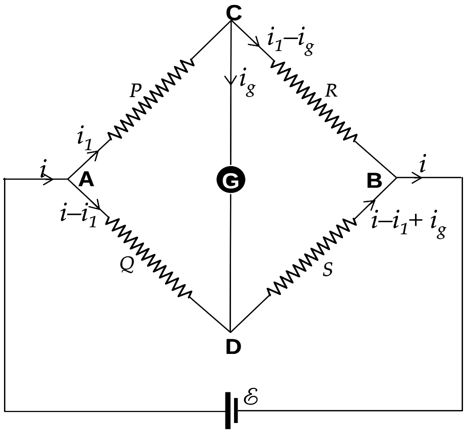


4. Deduction of *balancing condition* of a WSB:

Let total current *i* flows in to the Wheatstone's bridge.
According to junction rule, at points A, B, C and D:
At pint A, if i_1 flows through *P* then $i - i_1$ will flow through *Q*.
At point C, i_1 is divided in to i_g and $i_1 - i_g$, which flows through the galvanometer (with resistance G) and *R*, respectively.
At points D and B, currents combine, current through *S* is $i - i_1 + i_g$ and current *i* emerges at B.

Applying Kirchhoff's mesh rule for loop ACDA, $-i_1P - i_gG + (i - i_1)Q = 0$. ---- (i)

For loop CDBC, $-i_gG - (i - i_1 + i_g)S + (i_1 - i_g)R = 0$. ---------- (ii)



When the values of *P*, *Q*, *R* and *S* is so chosen that $i_g = 0$, the bridge is called '**balanced**'.
In this condition, the potentials of C and D are the same and equations (i) and (ii) take the form,

$$-i_1P + (i - i_1)Q = 0 \text{ and } -(i - i_1)S + i_1 R = 0$$

Solving them we have $\boxed{\frac{P}{Q} = \frac{R}{S}}$.

This is the **balancing condition** of WSB.