*scorepack* by Arvind Kumar Chapter 1- Electric Charge & Field

# **Chapter Overview**

Quantum of charge:  $e = 1.6 \times 10^{-19} \text{ C}$ .

1. Sharing of charge between two spherical conductors of radii  $R_1$  and  $R_2$  having total charge Q, when

braught to contact and then separated: 
$$Q'_1 = \frac{R_1 Q}{R_1 + R_2}$$
 and  $Q'_2 = \frac{R_2 Q}{R_1 + R_2}$ 

#### 2. Coulomb's Law:

(1) Magnitiue of Coulomb's force:

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$
 (vacuum);  $F = \frac{q_1 q_2}{4\pi\varepsilon_0 \varepsilon_r r^2}$  (in medium of d.e.c.  $\varepsilon_r$ )

(2) Vector form:  $\vec{F}_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \hat{r}_{12}, \ \vec{F}_{21} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \hat{r}_{21}$  hence  $\vec{F}_{21} = -\vec{F}_{21}$ 

(3) In terms of position vector of the charges:

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2(\vec{r}_1 - \vec{r}_2)}{\left|\vec{r}_1 - \vec{r}_2\right|^3}, \quad \vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2(\vec{r}_2 - \vec{r}_1)}{\left|\vec{r}_2 - \vec{r}_1\right|^3}$$

- 3. Relative permitivity or dielectric constant:  $\varepsilon_r = \frac{F_0}{F_m} = \frac{\varepsilon}{\varepsilon_0}$ .
- **4.** Electric Field:  $\vec{E} = \lim_{\Delta q \to 0} \frac{\Delta \vec{F}}{\Delta q}$ . **Unit:** NC<sup>-1</sup> or Vm<sup>-1</sup>. **Dimension:** [*MLA*<sup>-1</sup>*T*<sup>-3</sup>].

#### 5. Intensity of Electric Field

- A. Calculated by using Coulomb's law:
- (i) Due to a point charge:  $\vec{E} = -\frac{q}{4\pi \varepsilon_0 r^2} \hat{r}$ .

(ii) Due to a uniformly charged ring of radius on the axis:  $E = \frac{qx}{4\pi \varepsilon_0 (R^2 + x^2)^{3/2}}$ .

(iii) Due to a finite line of charge:  $E_x = \frac{\lambda}{4\pi \varepsilon_0 x} (\sin \theta_1 + \sin \theta_2), \quad E_y = \frac{\lambda}{4\pi \varepsilon_0 x} (\cos \theta_1 - \cos \theta_2)$ 

( $\theta_1$  and  $\theta_2$  are respectively, negative and positive angles made by the ends of the line at the point by the ends of the line charge.)

#### B. Calculated by applying Gauss's theorem:

- (i) Infinite line of charge:  $\vec{E} = \frac{\lambda}{2\pi \varepsilon_0 r} \hat{r}$
- (ii) Infinite sheet of charge:  $\vec{E} = \frac{\sigma}{2\varepsilon_0} \hat{n}$
- (iii) Outside a charged conductor:  $\vec{E} = \frac{\sigma}{\varepsilon_o} \hat{n}$

(iv) Between oppositely charged sheets or conducting planes:  $\vec{E} = \frac{\sigma}{\varepsilon_o} \hat{n}$ 

- (v) Charged hollow sphere:  $E = \frac{Q}{4\pi \varepsilon_0 r^2}$  (outside), E = 0 (inside).
- (vi) Charged solid sphere:  $E = \frac{Q}{4\pi \varepsilon_0 r^2}$  (outside),  $\vec{E} = \frac{Q\vec{r}}{4\pi \varepsilon_0 R^3}$  (inside).

### Physics Foundation

#### C. Due to an electric dipole:

(a) At an axial point :  $\vec{E} = \frac{2 \, \vec{p} \, r}{4\pi \, \varepsilon_0 \left(r^2 - l^2\right)^2}$ .

(b) At an equatorial point:  $\vec{E} = -\frac{\vec{p}}{4\pi \varepsilon_0 r^3}$ .

## 6. Superposition principle in electrostatics:

- The net force on  $q_0$  is the vector sum of all the forces,  $\vec{F} = \frac{q_0}{4\pi \varepsilon_0} \sum_{i=1}^n \frac{q_i}{r_{0i}^2} \hat{r}_{0i}$ .
- If  $q_i$  is the *i*<sup>th</sup> charged particle in the distribution of charges, then intensity of field at the point *P* will be

given as, 
$$\vec{E} = \frac{1}{4\pi \varepsilon_0} \sum_{i=1}^n \frac{q_i}{r_i^2} \hat{r}_i . \implies \vec{E} = \sum_{i=1}^n \vec{E}_i .$$

If charge is distributed continuously then net field at point  $E = \int dE$ .

- 7. Specific charge on a particle:  $\frac{q}{m}$ .
- 8. Acceleration of a charged particle in a field:  $a = \frac{qE}{m}$ .
- 9. Equation of trajectory of a charged particle in a uniform electric field when initial velocity v is along x axis and electric field is long y axis:  $y = \frac{qE}{2m}x^2$
- **10.** Electric dipole moment:  $\vec{p} = q\vec{L}$ . **Unit:** Cm, **Dimension:** [*ATL*].
- **11.** Torque on an electric dipole in a uniform electric field:  $\vec{\tau} = \vec{P} \times \vec{E}$  or  $\tau = PE \sin \theta$ .
- **12.** Electric flux:  $d\phi = \vec{E} \cdot d\vec{s}$ ; total flux  $\phi = \oint \vec{E} \cdot d\vec{s}$ .

Surface Unit: N m<sup>2</sup>/C or volt-m. **Dimensions:**  $[ML^3T^{-3}A^{-1}]$ .

**13.** Gauss's theorem:  $\phi = \frac{q}{\varepsilon_0}$ .

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