7. Capacitance & Capacitors

1. Capacitor:

***Capacitor*:** An arrangement of conductors to store charge, hence electrical energy is called a '*capacitor*'.

In practice, a capacitor consists of two conductors: one having charge and another earthed, separated by a small gap and filled with a dielectric (insulator). The system of these two conductors is called '*plates*'.

The charged and the earthed plates have equal and opposite charges, therefore the net charge on a capacitor is zero.

2. *Capacitance of a Capacitor:*

Capacitance of a Capacitor: Capacitance of a capacitor is the ratio of the charge on the +*ve* plate of the capacitor and potential difference between the plates.

$$\Rightarrow \boxed{C = \frac{q}{V_{Plates}}}.$$

Dependence of capacitance: Capacitance of a capacitor depends upon:

(i) Plates area A , **(ii)** Separation between the plates d and **(iii)** d.e.c. of the medium between the plates k, i.e.

$$C \propto A$$
$$\propto k$$
$$\propto \frac{1}{d}$$

3. Deduction of capacitance of parallel plate capacitor:

Two parallel plates of area A are arranged at a separation d.

Field between the plates is uniform and equal to $E = \frac{\sigma}{\varepsilon_0}$.

$\therefore$ Potential difference between the plates $V = Ed$.

$\Rightarrow V = \frac{\sigma}{\varepsilon_0} d = \frac{q}{A\varepsilon_0} d$, where q is the total charge on the charged plate.

$\therefore$ Capacitance $C = \frac{q}{V} = q / \frac{qd}{\varepsilon_0 A}$. $\boxed{\Rightarrow C = \frac{\varepsilon_0 A}{d}}$.

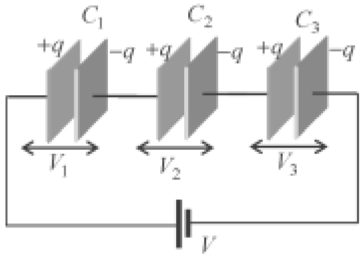
When an insulator of d.e.c. k is filled between the plates then $\boxed{C = \frac{\varepsilon_0 kA}{d}}$.

4. Grouping of a capacitor:

A combination of different capacitor is equivalent to a single capacitor. This is called a grouping. The simplest groupings are (i) series and (ii) parallel.

(a) Series grouping: When capacitors are grouped with alternate plates joined with one another, the grouping is called *series*. In such a connection, the charge on each capacitor is the same.

Let it be q. But p.d. across each capacitors will be different, let them be V_1, V_2, V_3 across C_1, C_2, C_3 respectively. $\therefore V_1 = \frac{q}{C_1}$, $V_2 = \frac{q}{C_2}$, and $V_3 = \frac{q}{C_3}$.



Total potential difference across combination is V, then,

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \Rightarrow \frac{V}{q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

Since q is total charge of the combination and V is pd across the combination, therefore $q/V = C$ is the equivalent capacitance of the combination.

$\therefore$ Equivalent capacitance of series combination of capacitors C_1, C_2, C_3 is $\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$.

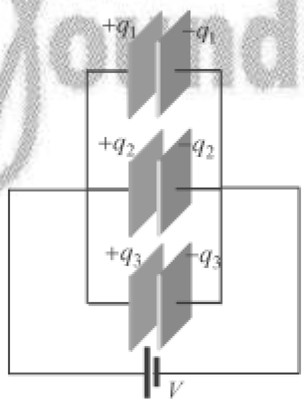
In general the equivalent capacitance $\frac{1}{C} = \sum_n \frac{1}{C_i}$, for n capacitors.

(b) Parallel combination: If charged plates of all capacitors are joined to one common point and earthed plates to another common point, the combination is called *parallel*.

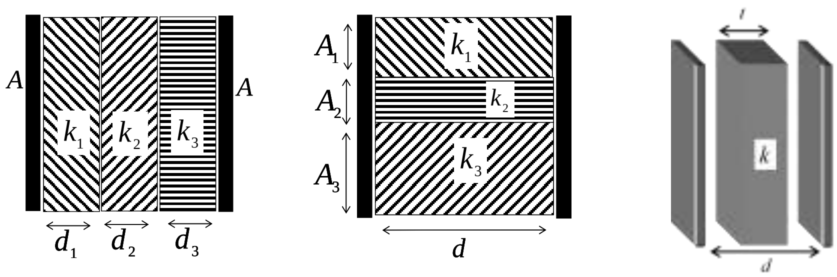
If C_1, C_2 and C_3 are capacitors joined in parallel then potential difference across each is the same. Let it be V. Let charges on them be Q_1, Q_2, and Q_3 respectively, then $Q_1 = C_1V$, $Q_2 = C_2V$ and $Q_3 = C_3V$.

$\therefore$Net charge on combination $Q = Q_1+Q_2+Q_3$. $\Rightarrow Q = V(C_1+C_2+C_3)$. $\Rightarrow \frac{Q}{V} = C_1 + C_2 + C_3$.

Since $Q/V = C$ is the equivalent capacitance of the combination, therefore, $\boxed{C = C_1 + C_2 + C_3}$.



5. Capacitors with mixed dielectrics:



(i) (ii) (iii)

(i) If dielectrics of d.e.c. k_1, k_2 and k_3 are arranged in full area of the plates with partial widths d_1, d_2 and d_3, the arrangement is series combination of three capacitors of capacitances $C_1=\frac{k_1\varepsilon_0 A}{d_1}$, $C_2=\frac{k_2\varepsilon_0 A}{d_2}$ and $C_3=\frac{k_3\varepsilon_0 A}{d_3}$. Hence equivalent capacitance is $\frac{1}{C_{eq}}=\frac{1}{\varepsilon_0 A}\left(\frac{d_1}{k_1}+\frac{d_2}{k_2}+\frac{d_3}{k_3}\right)$.

Equivalent dielectric constant: $C_{eq}=\dfrac{\varepsilon_0 A}{d\left(\frac{d_1}{k_1 d}+\frac{d_2}{k_2 d}+\frac{d_3}{k_3 d}\right)}$.

Therefore the equivalent dielectric constant, $\frac{1}{k_{eq}}=\frac{d_1}{k_1 d}+\frac{d_2}{k_2 d}+\frac{d_3}{k_3 d}$.

➤ **If** $d_1=d_2=d_3=d/3$ **then** $\frac{1}{k_{eq}}=\frac{1}{3}\left(\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_3}\right)$.

➤**If there are only two dielectrics then** $\frac{1}{k_{eq}}=\frac{1}{2}\left(\frac{1}{k_1}+\frac{1}{k_2}\right)$**,** or, $k_{eq}=\frac{2k_1k_2}{k_1+k_2}$. (Fig.1)

(ii) If dielectrics are arranged in full width and in partial plate areas A_1, A_2 and A_3, then the arrangement is a parallel combination of three capacitors of capacitances are:

$C_1=\frac{k_1\varepsilon_0 A_1}{d}$, $C_2=\frac{k_2\varepsilon_0 A_2}{d}$ and $C_3=\frac{k_3\varepsilon_0 A_3}{d}$.

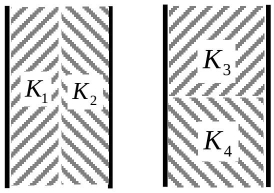
Hence equivalent capacitance is $C_{eq}=\frac{\varepsilon_0}{d}\left(k_1A_1+k_2A_2+k_3A_3\right)$.

Equivalent dielectric constant: $C_{eq}=\frac{\varepsilon_0 A}{d}\left(\frac{k_1A_1+k_2A_2+k_3A_3}{A}\right)$.

Therefore equivalent dielectric constant, $k_{eq}=\frac{k_1A_1+k_2A_2+k_3A_3}{A}$.

➤ **If** $A_1=A_2=A_3=A/3$ **then.** $k_{eq}=\frac{k_1+k_2+k_3}{3}$.

➤**If there are only two dielectrics then** $k_{eq}=\frac{k_1+k_2}{2}$. (Fig. 2)



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(iii) This is a special case of (i). The capacitor is the combination of an air capacitor of separation $d-t$ and another of t with dielectric k.

Hence $\frac{1}{C_{eq}}=\frac{1}{\varepsilon_0 A}\left(\frac{d-t}{1}+\frac{t}{k}\right)$. $\Rightarrow C_{eq}=\dfrac{\varepsilon_0 A}{\left(d-t+\frac{t}{k}\right)}=\dfrac{k\varepsilon_0 A}{k\,d-t\left(k-1\right)}$.

Equivalent dielectric constant: $C_{eq}=\dfrac{k\varepsilon_0 A}{d\left(k-\frac{t\left(k-1\right)}{d}\right)}$.

Therefore equivalent dielectric constant, $k_{eq} = \dfrac{k}{\left(k - \dfrac{t(k-1)}{d}\right)}$.

➢ **If** $t = d/2$ **then**. $k_{eq} = \dfrac{2k}{k+1}$.

6. Energy stored in a charged capacitor:

Let a capacitor be charged to a charge q and potential difference between the plates is V. If capacitance is C then $V = \dfrac{q}{C}$. Work done in transferring a further dq charge on the +ve plate from the negative plate, $dW = Vdq = \dfrac{qdq}{C}$.

$\therefore$ Total work done in depositing a total of charge Q on the +ve plate $w = \int_0^Q dw$.

$$\Rightarrow W = \int_0^Q \frac{qdq}{C} = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q \Rightarrow W = \frac{Q^2}{2C}.$$

This work done is stored in the capacitor as **potential energy** in the electric field between the plates.

Therefore potential energy of the capacitor is $U = \dfrac{Q^2}{2C}$ or $U = \dfrac{1}{2}CV^2$ or $U = \dfrac{1}{2}QV$.

7. Energy density of an electric field:

Energy stored in a parallel plate capacitor is given by the formula $U = \dfrac{1}{2}CV^2$.

This energy is stored in the electric field between the plates of a capacitor.

Considering the capacitor to be a parallel plate capacitor, $U = \dfrac{1}{2}\dfrac{\varepsilon_0 A}{d}V^2 = \dfrac{1}{2}\varepsilon_0 \dfrac{V^2}{d^2}Ad$.

$\dfrac{V}{d}$ is the uniform field E between the plates and Ad is the volume of the space between the plates in which uniform field exists.

Therefore, $\dfrac{U}{Ad} = \dfrac{1}{2}\varepsilon_0 E^2$, where $\dfrac{U}{Ad} = u$ =energy per unit volume in the field is called energy density of the field. Therefore the expression for the electric field energy is $u = \dfrac{1}{2}\varepsilon_0 E^2$.

This result is valid for a non-uniform electric field also. At a point where electric field is *E* the energy density will be $u = \dfrac{1}{2}\varepsilon_0 E^2$**.**

8. Energy loss in charging of capacitor:

Let a charge q be stored on a capacitor. Hence work done by the battery $W = qV$, where V is the pd between the terminals of the battery and battery is assumed to be ideal. Therefore energy supplied by the battery $U_{battery} = qV = CV^2$. Energy stored in the capacitor $U_{stored} = \dfrac{CV^2}{2}$.

Therefore, half part of the energy supplied is lost in instantaneous current flowing through the circuit during the charging process.

9. Energy loss in reconnection of two charged capacitors:

Supposed that two capacitors C_1 and C_2 are initially charged to pds V_1 and V_2.

Initial energy $U_i = \dfrac{1}{2}C_1V_1^2 + \dfrac{1}{2}C_2V_2^2$.

They are connected in parallel. Supposed across the combination the pd = V.

Therefore $\frac{q_1}{C_1} = \frac{q_2}{C_2} = V$. $\Rightarrow V = \frac{(q_1+q_2)}{C_1+C_2}$. (q_1+q_2) is the total initial charge $C_1V_1 + C_2V_2$.

Therefore $V = \frac{(C_1V_1+C_2V_2)}{C_1+C_2}$. Now final energy $U_f = \frac{1}{2}(C_1+C_2)V^2 = \frac{(C_1V_1+C_2V_2)^2}{2(C_1+C_2)}$.

Therefore, change in energy,

$$U_i - U_f = \frac{1}{2}\left(C_1V_1^2 + C_2V_2^2\right) - \frac{(C_1V_1+C_2V_2)^2}{2(C_1+C_2)}. \Rightarrow U_i - U_f = \frac{C_1C_2}{2(C_1+C_2)}(V_1-V_2)^2 > 0. \Rightarrow U_i > U_f.$$

Special case: If two capacitors are connected with reversed polarity then energy loss is:

$U_i - U_f = \frac{C_1C_2}{2(C_1+C_2)}(V_1+V_2)^2$. That is loss is greater.

10. Changes taking place when a dielectric slab is inserted between the plates of a parallel plate capacitor: (i) When plates are connected to a battery, (ii) when plates are charged but not connected to a battery:

(i) When plates are connected to a battery, and a dielectric slab is inserted between the plates of the capacitor then:

A. PD between the plates, hence the electric field between the plates remains constant.

B. Capacitance of the capacitor changes (increases) k times, where k is the dielectric constant of the capacitor.

C. Charge is directly proportional to the capacitance hence the charge also increases k times. That is new charge on the capacitor is $Q' = kQ$.

Therefore the charge increment is $\Delta Q = (k-1)Q$.

D. Induced charge on the surfaces of the dielectric is $Q_i = Q_{free}(1 - \frac{1}{k})$.

$\Rightarrow Q_i = kQ(1-\frac{1}{k}) = (k-1)Q = CV(k-1)$.

E. Since the energy is proportional to the capacitance then energy also increases k times. $U' = \frac{1}{2} kCV^2$.

(ii) If capacitor is charged and the cell is not connected and a dielectric slab is inserted between the plates of the capacitor then

A. Net charge on the capacitor remains the same.

B. Capacitance increases k times.

C. Field between the plates of the capacitor becomes $1/k$ times. As a result the potential difference between the plates also decreases $1/k$ times. $V' = V/k$.

D. Since energy of the capacitor is inversely proportional to the capacitance therefore the energy deceases $1/k$ times. $U' = \frac{1}{2} kC(V/k)^2$. $\Rightarrow U' = \frac{1}{2} CV^2/k$.

E. Since net charge on the capacitor remains the same, the induced charge on the dielectric surfaces is $Q_i = Q\ (1-\frac{1}{k})$.