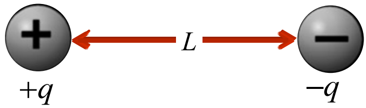
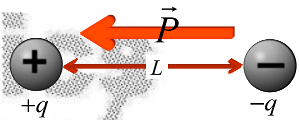
3. Electric Dipole

1. What is an *electric dipole*? Define *electric dipole moment*. *Write its SI unit*

► A system of two equal and opposite charges (+q) and (− q) separated by a small distance L is called an *electric dipole*. In nature, polar molecules are true electric dipoles.



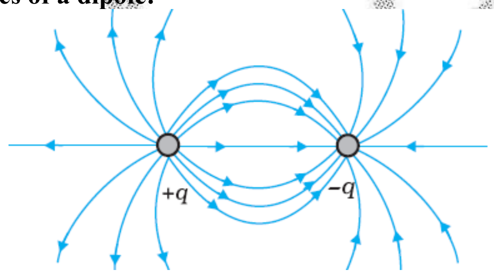
Dipole moment: Product of the magnitude of one of the charges q and the separation between the charges L is called *dipole moment* (P) of the electric dipole. $\therefore P = qL$.



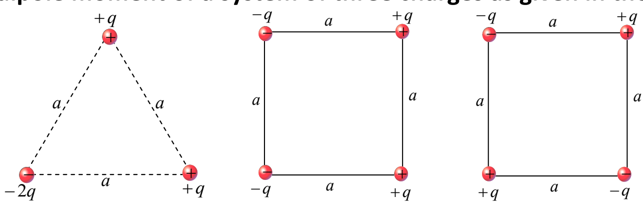
Dipole moment is a vector quantity with direction from –ve charge to +ve charge. The vector expression for dipole moment is given as $\vec{P} = q\vec{L}$. The vector $\vec{L}$ is *relative position vector* of + charge wrt −ve charge.

Unit: Cm. **Dimension:** [ATL].

Electric field lines of a dipole:



? Will there be a dipole moment associated with every system of charges? Calculate dipole moment of a system of three charges as given in the figure?



1 2 3

✓A dipole moment can be assigned only to that system of charges which has total charge zero. If a system of charge has total charge zero and sum of the moments of the charges is non zero then net moment represents dipole moment. That is for a system of charges $\sum q_i = 0$ and $\sum q_i r_i \neq 0$, then $\sum q_i r_i = \vec{P}$.

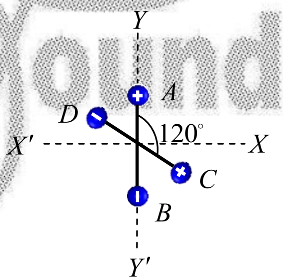
It is possible that for a system of charges $\sum q_i r_i$ also is zero, then that system of charge has centres of negative and positive charge coinciding and no dipole moment can be assigned to it.

(1) Taking $-2q$ as origin and the horizontal side as x –axis, the dipole moment can be calculated as: $\vec{p}_1 = qa\frac{\sqrt{3}}{2}(\sqrt{3}\hat{i} = \hat{j})$.

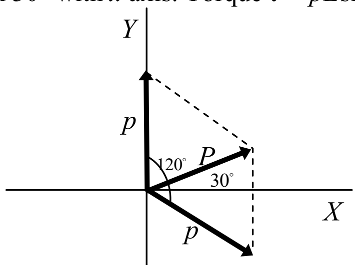
(2) Taking left lower corner as origin ($-q$) and horizontal side as x – axis, dipole moment is calculated as $\vec{p}_2 = qa\hat{i}$.

(3) Taking left lower corner as origin ($-q$) and horizontal side as x – axis, dipole moment is calculated to be 0.

? Two small identical electrical dipoles AB and CD, each of dipole moment '*p*' are kept at an angle of 120° as shown in the figure. What is the resultant dipole moment of this combination? If this system is subjected to electric field $\vec{E}$ directed along +X direction, what will be the magnitude and direction of the torque acting on this?



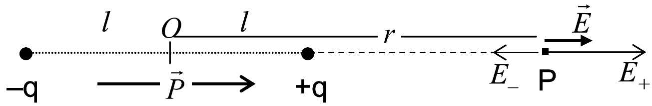
✓ The angle between the dipoles is θ = 120°. Therefore the resultant dipole moment $P = p$. direction 30° with x axis. Torque $\tau = pE\sin 30° = pE/2$.



2. Find the electric field intensity due to an electric dipole at an axial point.

► Let there be an electric dipole of charge magnitude q and separation between the charges $2l$, having centre at O.

Magnitude of the dipole moment is then $P = 2ql$. Let there be a point P on axial line at a distance r from the centre of the dipole.



Magnitude of field at point P due to the –ve charge is $E_- = \dfrac{q}{4\pi\varepsilon_0 (r+l)^2}$.

Field at P due to +ve charge is $E_+ = \dfrac{q}{4\pi\varepsilon_0 (r-l)^2}$.

The fields due to the positive and the negative charges are in opposite directions and E_+ is greater than E_-, the net field will be in the direction of E_+. Therefore the net field at P will be $E = E_+ - E_-$.

$$\Rightarrow E = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right).$$

Solving we get, $E = \dfrac{q4rl}{4\pi\varepsilon_0 (r^2 - l^2)^2} = \dfrac{2Pr}{4\pi\varepsilon_0 (r^2 - l^2)^2}$.

Since direction of net field is same as the direction of $\vec{P}$, in vector form the expression for the field will be $\vec{E} = \dfrac{2\vec{P}r}{4\pi\varepsilon_0 (r^2 - l^2)^2}$.

For a true dipole $l << r$, hence $\boxed{\vec{E} = \dfrac{2\vec{P}}{4\pi\varepsilon_0 r^3}}$.

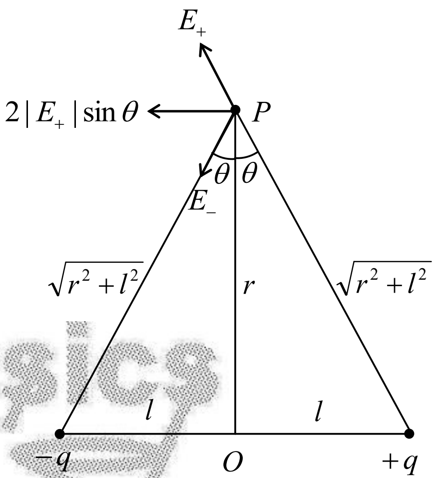
3. Find the electric field intensity due to an electric dipole at an equatorial point.

► If point P lies on the equatorial line at a distance r from the centre, then $|E_-| = |E_+| = \dfrac{q}{4\pi\varepsilon_0 (r^2 + l^2)}$.

Resultant of $\vec{E}_-$ and $\vec{E}_+$ is parallel to the axis of the dipole and opposite to $\vec{P}$.

Magnitude of the resultant electric field $E = \dfrac{2q}{4\pi\varepsilon_0 (r^2 + l^2)}\sin\theta$.

In figure, $\sin\theta = \frac{l}{\left(r^2+l^2\right)}$. $\therefore E = \frac{2ql}{4\pi\varepsilon_0\left(r^2+l^2\right)^{3/2}} = \frac{P}{4\pi\varepsilon_0\left(r^2+l^2\right)^{3/2}}$.



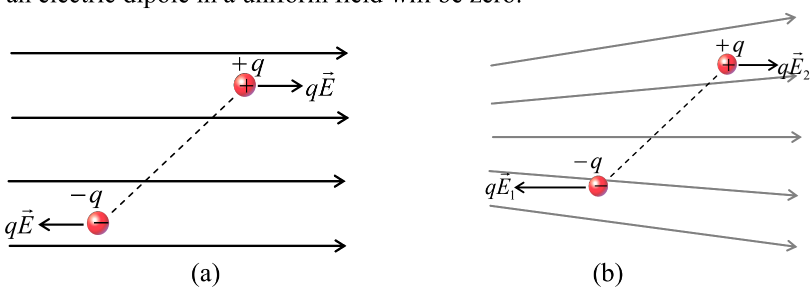
In vector form it is given as $\vec{E} = \frac{-\vec{P}}{4\pi\varepsilon_0\left(r^2+l^2\right)^{3/2}}$.

For a true dipole $l << r$, hence $\boxed{\vec{E} = -\frac{\vec{P}}{4\pi\varepsilon_0 r^3}}$.

Thus *field in axial direction is twice that in equatorial direction.*

❓Does a net force act on an electric dipole placed in a (a) uniform electric field? (b) Non-uniform electric field?

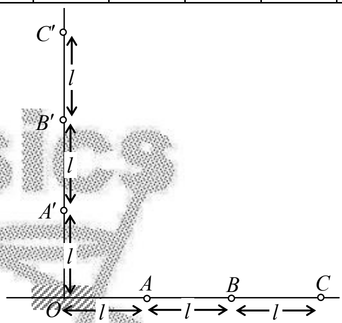
✓**(a)** In a uniform electric field, since at every point the field is the same, therefore the forces on the positive and the negative poles of the electric dipole will be the same in magnitude but exactly opposite in direction. So the net force on an electric dipole in a uniform field will be zero.



(b) In a non-uniform field, however, the electric field is different at different points therefore forces on the two poles of the dipole will be different, both in magnitude and direction giving a non-zero net force on the dipole.

? The following data was obtained for the dependence of the magnitude of electric field, with distance, from a reference point *O*, due to the charge distribution in the shaded region.

Field Points	A	B	C	A′	B′	C′
Electric field intensity	E	$E/8$	$E/27$	$E/2$	$E/16$	$E/54$

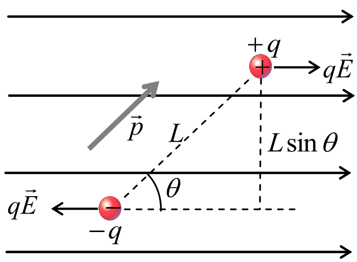


✓ The data shows two important features: (i) Field is inversely proportional to the cube of the distance and (ii) the field at the points on A′, B′, C′ are half of the corresponding values at A, B, C. These two features are of the field due to a dipole, A-B-C being the axial points and A′-B′-C′ being the equatorial points. Therefore the charge distribution at *O* is electric dipole.

---

4. Find torque on an electric dipole placed in a uniform electric field.

►Supposed an electric dipole with dipole moment $P = qL$, is placed in an uniform electric field of intensity E, with the axis of dipole making angle θ with the direction of $\vec{E}$.



The forces acting on the poles (on the negative and the positive charges) are in opposite directions and equal to $F = qE$ on each. But the distance between the lines of action of forces is $L\sin\theta$, therefore there is a net torque acting on the dipole with magnitude $\tau = qEL\sin\theta$. $\Rightarrow$ $\tau = PE\sin\theta$.

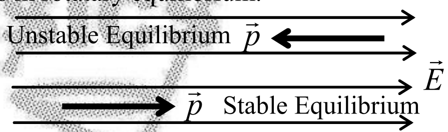
Since P and E both are vectors and θ is the angle between them, therefore $\tau = \left|\vec{P}\times\vec{E}\right|$, or $\vec{\tau} = \vec{P}\times\vec{E}$.

When θ = 0, and θ = 180° the torque = zero and for θ = 90° torque is maximum, $\tau = PE$.

The torque tends to rotate the dipole clockwise hence ***tends to align the dipole in the direction of the field***.

5. In which orientation the equilibrium of the electric dipole is stable? Discuss the equilibrium of a dipole in uniform electric field.

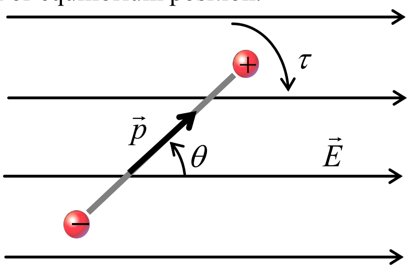
► For the angles θ = 0 and θ = 180° or when the dipole is either parallel or anti-parallel to the electric field the torque is zero on the dipole, hence the dipole is in translatory as well as in rotatary equilibrium.



(i) When dipole is parallel to the field it is in stable equilibrium.
(ii) When dipole is anti-parallel to the field it is in unstable equilibrium.

6. If an electric dipole of dipole moment *p* and moment of inertia *I* is in stable equilibrium position in a uniform electric field *E*. It is slightly rotated and released, then prove that it execute SHM. What is time period of this oscillation?

►Supposed the dipole moment vector is making an angle θ with field vector at any moment during the motion, after it is released. The restoring torque which tries to make θ zero, is $\tau = pE\sin\theta$. If α is the angular acceleration of the dipole then, $\tau = I\alpha = -pE\sin\theta$. Minus sign indicates that angular acceleration is always opposite to the angular position or displacement of the dipole with reference to the mean or equilibrium position.



If θ is very small, so that $\sin\theta \to \theta$ then $I\alpha = -pE\theta$ or $\alpha = -\frac{pE}{I}\theta$.

This implies that, $\alpha \propto -\theta$, the motion of the dipole is an SHM for small oscillations.

The square of angular acceleration of motion is $\omega^2 = \frac{qE}{I}$.

Hence, $\omega = \sqrt{\frac{qE}{I}}$. The time period is $T = 2\pi\sqrt{\frac{I}{qE}}$.