1. Electric Potential Energy

1.Electric potential energy:

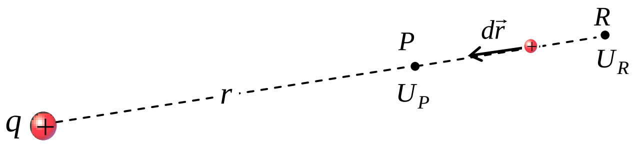
Energy of electrostatic interaction is called **electrical potential energy**.

Change in electric potential energy of a system of charged particles is measured by the work done by external agent or negative of work done by electrical forces in making the system.

Supposed in the electric field of charge q, a small charge Δq is displaced by $d\vec{r}$, then work done by external agent is $dW_{ext} = \vec{F}_{ext} \cdot d\vec{r}$. Therefore change in potential energy, $dU = dW_{ext} = \vec{F}_{ext} \cdot d\vec{r}$.

[Or, $dU = -\vec{F}_{ele} \cdot d\vec{r}$, because work done by electrical force is equal and opposite to that done by an external agent.]

Supposed the external force $\vec{F}_{ext}$ takes the charge Δq form a point R to another point P in the field so that charge does not accelerate and moves slowly with a constant speed.



Then the total work done in taking the body from R to P will be : $W_{RP} = \int_R^P \vec{F}_{ext} \cdot d\vec{r} = -\int_R^P \vec{F}_{ele} \cdot d\vec{r}$.

Since, $W_{RP} = U_P - U_R$, therefore, $U_P - U_R = \int_R^P \vec{F}_{ext} \cdot d\vec{r} = -\int_R^P \vec{F}_{ele} \cdot d\vec{r}$.

Hence the change in potential energy is **positive work done by an external force** equal and opposite to the electric field, or, **negative of the work done by the electric field**.

Alternately:

The potential energy of a charged particle decreases if positive work is done by the electric field and increases if the external agent does positive work.

2. ‘Absolute potential energy at a point is arbitrary to within an additive constant’:

Absolute potential energy at a point can not be determined uniquely and it is arbitrary to within an additive constant.

That is, **absolute potential energy of a charged particle can be increased or decreased by same amount at every point.**

Supposed at points P and R potential energies of a particle are U_P and U_R respectively. Then energies may be given as $U_P + \alpha$ and $U_R + \alpha$, also, where α is a constant negative or positive. But the difference $\Delta U = U_P - U_R$ remains the same which is significant and is equal to work done.

3.Electric potential energy of a charged particle at a point in an electric field:

Electric potential energy is defined as:

“The work done by an external force (equal and opposite to the electric force) in bringing a charged particle q from infinity to a point is equal to the potential energy of the charged particle q at that point, provided the potential energy at infinity is taken to be zero.”

4.Electric potential energy of a two-charge system:

The negative of work done by the electrical forces in bringing two charges at a finite separation from infinity is the potential energy of a “two charge system”.

In other words, the potential energy of one of the charges in the field of the other is the potential energy of the system of the two charges.

Therefore the potential energy of the system of two charges q_1 and q_2, which are at a separation r will be: $U(r) = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r}$.

5.Calculation of potential energy of a two charge system:

By the definition of the potential energy, $U(r)=\int_{\infty}^{r}\vec{F}_{ext}\cdot d\vec{r}=-\int_{\infty}^{r}\vec{F}_{electric}\cdot d\vec{r}$.

Force on a charged particle q_2 in an electric field of source particle q_1 at a distance r is $\vec{F}=\frac{q_1q_2}{4\pi\varepsilon_0 r^2}\hat{r}$.

If q_0 is brought near to the charge q by a displacement dr against the electric force then,

$\vec{F}_{ext}\cdot d\vec{r}=-\frac{q_1q_2}{4\pi\varepsilon_0 r^2}dr$. Therefore, $U(r)=-\frac{q_1q_2}{4\pi\varepsilon_0}\int_{\infty}^{r}\frac{dr}{r^2}$. $\Rightarrow U(r)=\frac{q_1q_2}{4\pi\varepsilon_0 r}$.

Sign of the potential energy: The potential energy is positive if both the charges are of the same sign and it is negative if the two charges are of the opposite sign. The sign of the potential energy signifies that whether the field has done positive work or some external force has done positive work against the electric field in bringing the charges from the infinity to the final point.

6.Electrostatic potential energy of a system of many charges system:

Energy possessed by a system of electrically interacting charges, which depends on the configuration of the charge-system, is called ‘electric potential energy of the system of charges’.

This is equal to the *negative of work done by the electrical forces in assembling the charges or creating the system of charges.*

In a system of many charges, the sum of PE of all *two-charge systems* is the total electrical potential energy of the *many charge system.*

If there are n charges in a system, then number of *two charge systems* will be ${}^{n}C_2=\frac{n!}{2!(n-2)!}$.

The sum of potential energy of these all two charge systems is the net potential energy of the system of n particles.

Mathematically, $U=\sum_{i\neq j,\ ij=ji}\frac{q_iq_j}{4\pi\varepsilon_o r_{ij}}$. Or, $U=\frac{1}{2}\sum_{i\neq j}\frac{q_iq_j}{4\pi\varepsilon_o r_{ij}}$.