2. Electrostatic Potential

1. Define *potential* in an electric field. Give its unit and dimensions.

Definition I:

Potential energy per unit charge at a point in an electric field is called potential at that point. If a test charge q has a potential energy U at a point then electric potential at that point will be $V = U/q$.

Definition II:

Work done per unit charge by an external force (or negative of work done by electric force) in bringing a test charge from infinity to a point in an electric field, is called the ***potential*** *at that point.*

Unit: By definition, unit is joule/coulomb (**J/C**) or **volt** (V).

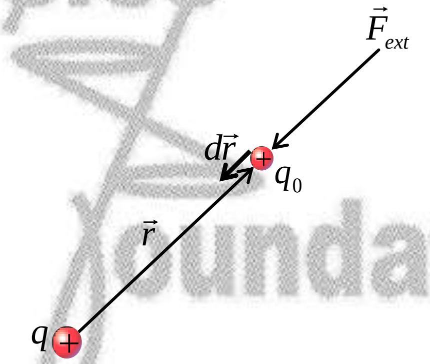
Dimension: $[ML^2T^{-3}A^{-1}]$.

Potential is a scalar quantity. It can be negative or positive, like work done or potential energy. Potential at a point due to a positive source charge is positive while due to a negative source charge it is negative.

2. Calculation of the 'electric potential' at a point in an electric field due to a point source charge *q*:

Supposed a point source charge q is located at the origin and a test charge q_0 is located at a position $\vec{r}$. If it is displaced by an external force (equal in magnitude of the electric force) by $d\vec{r}$ towards the origin, the change in potential energy will be $dU = \vec{F}_{ext} \cdot d\vec{r} \Rightarrow dU = -\frac{qq_0}{4\pi\varepsilon_0 r^2} dr$

(Because dr is decrease in the distance r, or dr is the negative displacement).

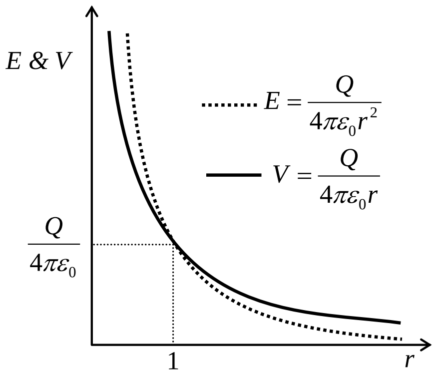


Therefore change in potential in bringing the test charge from infinity to a point having position vector r will be,

$$V(r) - V(\infty) = -\int_{\infty}^{r} \frac{q}{4\pi\varepsilon_0 r^2} dr = -\frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{\infty}^{r} = \frac{q}{4\pi\varepsilon_0 r}.$$

If we assume $V(\infty) = 0$, then potential at position r from the source charge is $V(r) = \frac{q}{4\pi\varepsilon_0 r}$.

3. Comparison of the graph for *E* and *V* for a point charge:



Electric field due to a point chare Q varies as $E(r) = \dfrac{Q}{4\pi\varepsilon_0 r^2}$

Electric potential due to a point chare Q varies as $V(r) = \dfrac{Q}{4\pi\varepsilon_0 r}$.

That is $E(r) \propto \dfrac{1}{r^2}$ and $V(r) \propto \dfrac{1}{r}$. Therefore the graph of E will be steeper. At r = 1 unit both the graphs will intersect because E and V both have same numerical value $E(r) = \dfrac{Q}{4\pi\varepsilon_0}$.

4. Potential difference between the two points:

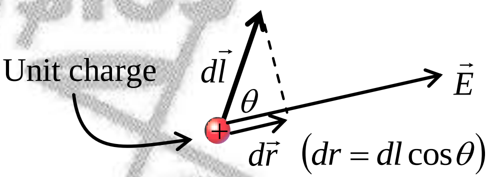
Change in potential will be, $V_P - V_R = \dfrac{W_{ext}}{q} = -\dfrac{W_{ele}}{q}$ = work done by external agent or negative of the work done by electric force per unit charge in bringing a test charge from a point R to P.

5. Relation between *field* and *potential* at a point in an electric field:

Supposed electric field at a point is $\vec{E}$. Assumed that a unit charge is displaced by a displacement $d\vec{l}$ over which the magnitude of field does not change considerably.

Therefore *work done by the electric force on the unit charge* = $\vec{E} \cdot d\vec{l}$.

The change (fall) in the potential over displacement dl will be given by $\vec{E} \cdot d\vec{l} = -dV$. …………..(i)



Now, $\vec{E} \cdot d\vec{l} = Edl\cos\theta = Edr$ (supposed)…………….(ii)

Here $dr = dl\cos\theta$ is the component of $d\vec{l}$ in the direction of $\vec{E}$.

Therefore by (i) and (ii), $E\,dr = -dV$. $\Rightarrow$ $E = -\dfrac{dV}{dr}$. In vector form it is written as $\vec{E} = -\dfrac{d\vec{V}}{dr}$.

This is the relation between field and potential.

The quantity $\dfrac{d\vec{V}}{dr}$ is called potential gradient (change of potential per unit distance in the direction of the field). Therefore, at any point, the intensity of the field is defined as *negative of the potential gradient at that point* and unit of electric field is given as **V/m** also.

Significance of the relation: Since field is negative of the rate of change in potential at a point, this relation implies that *at any point in an electrostatic field, the potential decreases in the direction of the field vector*. Greater the gradient, stronger is the field.

6. Electrostatic potential energy of a system of many charges in an electric field:

If a system of many charges is placed in an external electric filed, apart from the energy of the system each of the charged particle will possess potential energy due to the electric field.

Supposed there is a two-charge system, having charges q_1 and q_2, located at the positions A and B distance d apart in an external electric field where potentials are V_A and V_B respectively.

Then, the potential energy due to the external field will be: $U_{ext} = q_1V_1 + q_2V_2$.

Internal potential energy of two-charge system: $U_{\text{int}} = \dfrac{q_1q_2}{4\pi\varepsilon_0 d}$.

Hence total energy $U = \frac{q_1q_2}{4\pi\epsilon_0 d} + (q_1V_1 + q_2V_2)$.