3. Potential Due to Dipole

1. Potential due to a dipole:

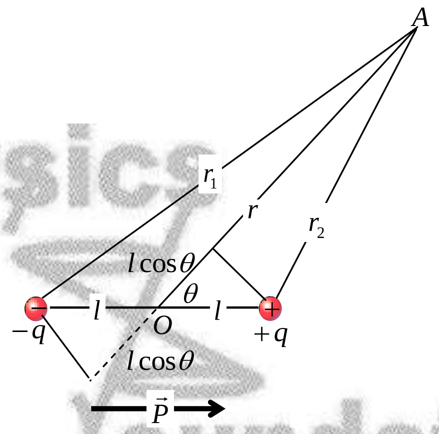
Let r_1 and r_2 are the distances of A from negative and the positive charge respectively. Therefore the potential at A due to the negative and positive charge are $V_- = -\frac{Kq}{r_1}$ and $V_+ = \frac{Kq}{r_2}$, respectively.

Since $l << r$, $r_1 = r + l\cos\theta$ and $r_2 = r - l\cos\theta$.

Therefore net potential at the point A is $V = V_- + V_+ = Kq\left[\frac{1}{r - l\cos\theta} - \frac{1}{r + l\cos\theta}\right]$.

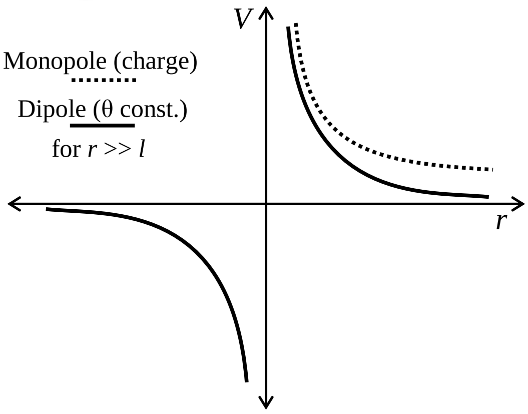
$\Rightarrow V = Kq\left[\frac{2l\cos\theta}{r^2 - l^2\cos^2\theta}\right]$.

Neglecting $l\cos\theta$ in the denominator, $V = K\frac{P\cos\theta}{r^2}$.



Since θ is the angle between $\vec{P}$ and the unit position vector $\hat{r}$ of point A w.r.t. the center of the dipole O; therefore, in vector form the relation can be given as $V = K\frac{\vec{P}\cdot\hat{r}}{r^2}$.

Graph for ideal and real dipoles are as follows:



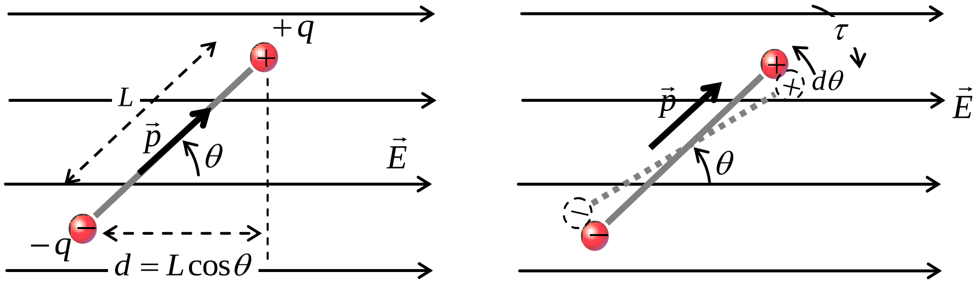
2. Find the potential energy of an electric dipole in an electric field.

Supposed that a dipole having dipole moment vector $\vec{P}$ is placed in a uniform electric field $\vec{E}$ making an angle θ with $\vec{E}$. Then torque on the dipole trying to bring $\vec{P}$ parallel to $\vec{E}$ will be $\tau = PE\sin\theta$. If the dipole is rotated by a small angle $d\theta$, by applying a torque equal to that applied by electric field in opposite direction, then the work done by the external torque will be: $dW_{ext} = PE\sin\theta d\theta$.

Net work done in rotating the dipole from a position , assumed to be zero potential energy position, to the final position θ, will be:

$W_{ext} = \int_{\pi/2}^{\theta} PE\sin\theta\, d\theta = U$. $\Rightarrow U = PE[-\cos\theta]_{\pi/2}^{\theta}$. Or $U = -PE\cos\theta$.

In vector form, $U = -\vec{p}\cdot\vec{E}$.



The internal or self-potential energy of the dipole will be: $U_{self} = -\dfrac{q^2}{4\pi\varepsilon_0 L}$.

Supposed the poles with charges $\pm q$ are located at points A and B, where potentials are V_1 and V_2 respectively. Therefore, total potential energy of dipole in the electric field will be:

$$U = -\vec{p}\cdot\vec{E} - \frac{q^2}{4\pi\varepsilon_0 L}.$$

Since the potential energy is defined as the difference of energy, then constant term (of internal potential energy) has no significance and the equation reduces to $\boxed{U_{dipole} = -\vec{P}\cdot\vec{E}}$.