2. Magnetic Field due to Current

1.Oersted's experiment:

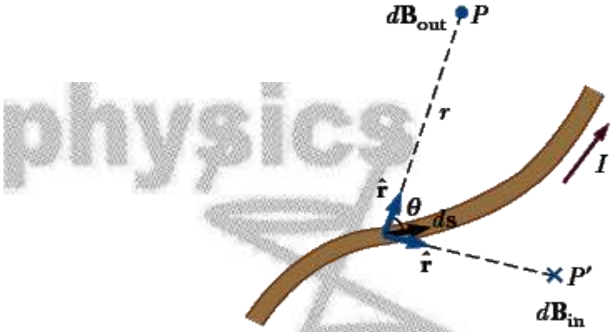
▶ Oersted found that if a magnetic needle (compass) is brought near a wire carrying current, it deflects. When the current flows in the opposite direction, deflection too is in reverse direction.
So, he concluded that current i.e. a moving charge produces a magnetic field.

2.*Current element*:

▶ Product of magnitude of current *i* and an elementary length of current path *dl* is defined as current element. Therefore current element = *idl*. **Unit**: A m. It is taken as a vector quantity having direction along the direction of current.

3.Biot - Savart Law:

▶ According to Biot-Savart law (after the name of Jean Baptist Biot, 1774-1862 and Felix Savart, 1781-1841), "when a current flows through a conductor, the magnetic field due to a current element $i\vec{dl}$, at a position $\vec{r}$ with respect to the current element is given by $d\vec{B} = \frac{\mu i\vec{dl} \times \hat{r}}{4\pi r^2}$, where μ is a constant called '*permeability*' of the medium in which the field is measured". For vacuum, it is called '*permeability of vacuum*' denoted as μ_0 having value $4\pi \times 10^{-7}$ Hm^{-1}.



4.Direction of the magnetic field due to a current element :

▶ To determine the direction of the magnetic field due to a straight current carrying element, the **right hand thumb rule** is applied. According to the right hand rule, if a current carrying conductor is held by the right hand so that the thumb directs in the current, then the direction of *curling* of the fingers indicates the direction of the magnetic field round the conductor.

❓ An electric current flows in a wire from north to south. What will be the direction of the magnetic field due to this wire at a point east of the wire? West of the wire? Vertically above of the wire? Vertically below of the wire?
Ans: Upward, downwards, westwards and eastwards.

❓ Do magnetic forces obey Newton's third law? Verify for two current elements $i\vec{dl_1} = idl\hat{i}$ located at the origin and $i\vec{dl_2} = idl\hat{j}$ located at (0, *R*, 0). Both carry current *I*.

<u>Ans</u>: No. Force due to $i\vec{dl_2}$ on $i\vec{dl_1}$ is zero. Force due to $i\vec{dl_1}$ on $i\vec{dl_2}$ is non-zero.

5.Relative permeability of a medium:

▶ Strength of magnetic field is different in different media for same $id\vec{l}$ and $\vec{r}$. The constant determining the strength of field is called *relative permeability* of a medium denoted as μ_r. This is the ratio of field in medium is to field in vacuum.

$\therefore \mu_r = \dfrac{dB(\text{medium})}{dB(\text{vaccum})} \Rightarrow \mu_r = \dfrac{\mu}{\mu_0}$ $\boxed{\therefore \mu_r = \mu_0 \mu}$.

Relative permeability is the characteristic of a medium.

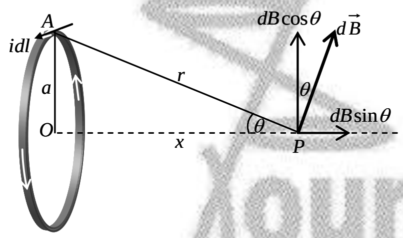
6.Expression for $\vec{B}$ on the axis of a circular current loop (or coil):

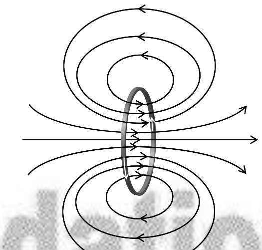
▶ Let there be a point *P* on the axis of a circular current carrying loop of radius *a*. Distance of *P* is *x* from the centre *O* and current through the loop is *i*. Let there be an element *idl* at *A*. *AP* = *r*. Position of *P* (w.r.t. *A*) $\vec{r}$ is perpendicular to $id\vec{l}$. Hence field at *P* due to *idl* is $\overrightarrow{dB} = \dfrac{\mu_0}{4\pi}\dfrac{id\vec{l}\times\hat{r}}{r^2}$.

$\Rightarrow dB = \dfrac{\mu_0 idl}{4\pi r^2}$. (directed $\perp_{er}$ to *AP*).

If angle *APO* = *θ*, then component of $\overrightarrow{dB}$ along and perpendicular to the axis is $dB\sin\theta$ and $dB\cos\theta$ respectively. $dB\cos\theta$ will be annulled by the component of the field due to counterpart element, hence, effective field is $B = \oint dB\sin\theta$, along the axis.

$\Rightarrow B = \oint \dfrac{\mu_0 idl}{4\pi r^2}\sin\theta . \Rightarrow B = \int_0^{2\pi a} \dfrac{\mu_0 idl}{4\pi r^2}\sin\theta = \dfrac{\mu_0 i}{4\pi r^2}[l]_0^{2\pi a} . \Rightarrow B = \dfrac{\mu_0 i\sin\theta}{4\pi r^2}2\pi a . \Rightarrow B = \dfrac{\mu_0 ia}{2r^2}\sin\theta$.





Field of a single loop

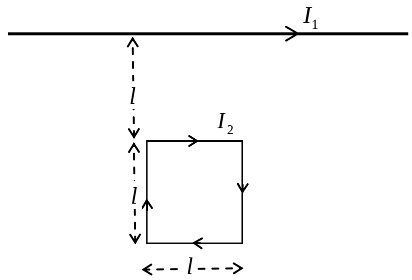
$\because \sin\theta = \dfrac{a}{r}$ and $r = \sqrt{a^2 + x^2}$, therefore $\boxed{B = \dfrac{\mu_0 ia^2}{2(a^2+x^2)^{3/2}}}$.

If there is a coil of number of turns *N* then the field formula will become:

$$\boxed{B = \dfrac{N\mu_0 ia^2}{2(a^2+x^2)^{3/2}}}$$

Special Case: If *x* = 0, for the centre of the loop, $\boxed{B = \left(\dfrac{\mu_0 i}{2a}\right)}$.

? In the given figure this loop is placed in a horizontal plane near a long straight conductor carrying a steady current I_1 at a distance *l* as shown. Give reasons to explain that the loop will experience a net force but no torque. Write the expression for this force acting on the loop.



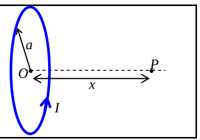
Ans: Magnetic moment $\vec{m} = I_2 l^2 \hat{n}$. $\hat{n}$ is the unit vector perpendicular to the plane of the loop directed inwards.
The field due to current I_1 is parallel to the magnetic moment of the loop. Therefore there will be no torque acting on the loop.
The current I_2 in the nearer side of the loop is parallel to I_1 while in the farther side it is opposite to I_1. The attractive force between the parallel current thus exceeds repulsion between the antiparallel currents. Therefore there will be a net attractive force on the loop.

The attractive force between the nearer side of the loop and the wire is $F_1 = \frac{\mu_0 I_1 I_2 l}{2\pi l} = \frac{\mu_0 I_1 I_2}{2\pi}$.

The attractive force between the nearer side of the loop and the wire is $F_2 = \frac{\mu_0 I_1 I_2 l}{2\pi(2l)} = \frac{\mu_0 I_1 I_2}{4\pi}$.

Therefore net force: $F = F_1 - F_2 = \frac{\mu_0 I_1 I_2}{2\pi} - \frac{\mu_0 I_1 I_2}{4\pi}$. $\Rightarrow F = \frac{\mu_0 I_1 I_2}{4\pi}$.

❓ A student records the following data for the magnitudes (B) of the magnetic field at axial points at different distances x from the centre of a circular coil of radius a carrying a current I. Verify (for any two) that these observations are in good agreement with the expected theoretical variation of B with x.

$x \rightarrow$	$x = 0$	$x = a$	$x = 2a$	$x = 3a$	
$B \rightarrow$	B_0	$0.25\sqrt{2}B_0$	$0.039\sqrt{5}B_0$	$0.010\sqrt{10}B_0$	

Ans: Theoretical variation of B with x is given as $B(x) = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}$.

At $x = 0$, $B = B_0$. Therefore $B(x) = \frac{B_0 a^3}{(a^2 + x^2)^{3/2}}$. Placing $x = a$ we have, $B(a) = \frac{B_0}{(2)^{3/2}} = 0.25\sqrt{2}B_0$.

Hence first observation is verified.

Placing $x = 2a$ we have, $B(2a) = \frac{B_0}{(5)^{3/2}} = 0.04\sqrt{5}B_0$. Hence second observation is correct within the limit of accuracy.
Therefore, 2nd observation is verified.

Placing $x = 3a$ we have, $B(3a) = \frac{B_0}{(10)^{3/2}} = 0.010\sqrt{10}B_0$. Hence 3rd observation is also verified.

❓ A circular coil of N turns and radius R carries a current I. It is unwound and rewound to make another coil of radius $R/2$, current I remaining the same. Calculate the ratio of the magnetic moment of the new coil and the original coil.

Ans: Initial magnetic moment $M_1 = N\pi R^2 I$.
When radius is halved the number of turns in the coil becomes twice.

Therefore, final magnetic moment $M_2 = 2N\pi \frac{R^2}{4} I$. $\Rightarrow \frac{M_2}{M_1} = \frac{1}{2}$.

❓ A current carrying loop consists of 3 identical quarter circles of radius R, lying in the positive quadrants of the x-y, y-z and z-x planes with their centres at the origin, joined together. Find the direction and magnitude of B at the origin.

Ans: $\vec{B} = \frac{1}{4}(\hat{i} + \hat{j} + \hat{k}) \frac{\mu_0 I}{2R}$.

❓ You are facing a circular wire carrying an electric current. The current is clockwise as seen by you. Is the field at the centre coming towards you or going away from you?
Ans: Going away.

❓ A long, straight wire carries a current. Is Ampere's law valid for a loop that does not enclose the wire? That encloses the wire but is not circular?
Ans: For the loop that doesn't encloses any current, line integral of the magnetic field is zero round the loop but the law is valid. The Ampere's law is valid for any shape of the loop.

❓ A straight wire carrying an electric current is placed along the axis of a uniformly charged ring. Will there be a magnetic force on the wire if the ring starts rotating about the wire? If yes, in which direction?

Ans: No! The rotating charged ring is like a current loop that creates field in the direction of the current, that is, along the wire, therefore no force will act on the wire.

‽ Two wire carrying equal current *i* each, are placed perpendicular to each other, just avoiding a contact. If one wire is held fixed and the other is free to move under magnetic forces, what kind of motion will result?

Ans: The movable wire will experience a torque which tends to rotate the wire to make it parallel to the fixed wire.

‽ Two proton beams going in the same direction repel each other whereas two wires carrying current in the same direction attract each other. Explain.

Ans: In first case electrostatic force exceeds magnetic force. But in the latter case there is no electrostatic force between the current carrying wires, and magnetic force causes the two wires attract each other.

7.Circular current loop is equivalent to a magnetic dipole.

▶The magnetic field on the axis of a tiny magnetic dipole, at a distance x from the center of the dipole is given as $\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{P}_m}{x^3}$. The length of the dipole is $l << x$.

Field due to a circular current loop on its axis is given as $\boxed{B = \frac{\mu_0 i a^2}{2(a^2 + x^2)^{3/2}}}$.

If we consider magnetic field at distance x from the center of the current loop, where radius of the loop $a << x$, then we can neglect a in the denominator and introduce the 2π in the numerator and denominator. Therefore the expression becomes, $B = \frac{\mu_0 2i(\pi a^2)}{4\pi\ x^3}$.

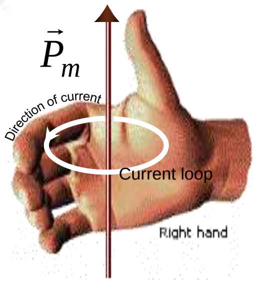
If we assume $i(\pi a^2)$ equivalent to the magnetic dipole moment P_m having direction along $\vec{B}$, then the expression becomes $\vec{B} = \frac{\mu_0 2\vec{P}_m}{4\pi\ x^3}$.

Hence a current loop can be considered to be a tiny magnet located at its center, along the axis, with magnetic dipole moment $\vec{P} = i\vec{A}$, where $\vec{A}$ is the area of loop.

8.Define magnetic dipole moment of a current loop.

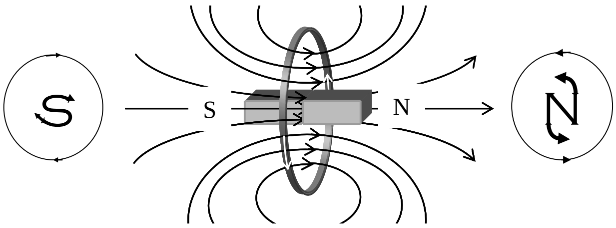
▶ *Product of area of a current loop and the current through it is magnetic dipole moment of the current loop*.

It is a vector quantity having direction along the normal to the area of the plane or parallel to magnetic field produced by the loop. $\Rightarrow \vec{P}_m = i\vec{A}$.



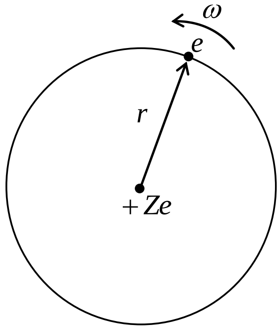
The current loop can be considered to be a tiny magnet located at its center, along the axis, having the direction of the dipole moment pointing along the direction of the magnetic field.

A current loop having current clockwise if viewed from the front has its South Pole in front and North Pole in back. If the current is anticlockwise, the South Pole is in back and the North Pole is in front, as shown in the figure.



9.Magnetic dipole moment of an electron orbiting around the central nucleus of an atom of atomic number Z:

▶Magnetic moment of an electron is μ = orbital current × area of the orbit.



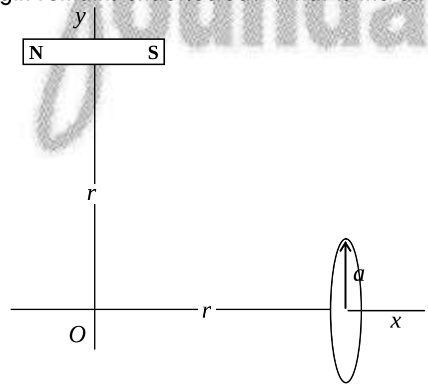
Orbital current $I = e\times f = \frac{e\omega}{2\pi}$. Area = πr^2. Therefore $\mu = \frac{e\omega r^2}{2}$.

For the stability of the orbit, $m\omega^2 r = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$, where Z is the number of protons in the nucleus. $\Rightarrow$ $\omega r^2 = \sqrt{\frac{Ze^2 r}{4\pi\varepsilon_0 m}}$. Therefore $\mu = \sqrt{\frac{Ze^4 r}{16\pi\varepsilon_0 m}}$.

❓ Two circular loops of radii r and $2r$, have currents I and $I/2$ flowing through them, in clockwise and anticlockwise sense respectively. If their equivalent magnetic moments are $\vec{M}_1$ and $\vec{M}_2$ respectively. State the relation between $\vec{M}_1$ and $\vec{M}_2$.

Ans: $\vec{M}_2 = -2\vec{M}_1$.

❓ A small magnet of magnetic moment *m* is placed at a distance *r* from the origin O with its axis parallel to *x*-axis as shown. A small coil of one turn is placed on the x – axis at the same distance from the origin, with the axis of the coil coinciding with the *x*-axis. For hat value of current in the coil does a small magnetic needle kept at the origin remains undeflected? What is the direction of the current in the coil?



Ans: Let the magnetic field due to the dipole at O be $\vec{B}_1$ and that due to the loop be $\vec{B}_2$. For zero field at O, $\vec{B}_1 + \vec{B}_2 = 0$. $\Rightarrow$ $\vec{B}_1 = -\vec{B}_2$. $\Rightarrow$ $|\vec{B}_1| = |-\vec{B}_2|$.

Now $|\vec{B}_1| = \frac{\mu_0 m}{4\pi r^3}$ and $|\vec{B}_2| = \frac{\mu_0 I a^2}{2r^3}$. Therefore, $\frac{\mu_0 m}{4\pi r^3} = \frac{\mu_0 I a^2}{2r^3}$.

Hence current in loop will be $I = \frac{m}{2\pi a^2}$.

$\vec{B}_1$ will be along positive *x*-axis, hence the direction of $\vec{B}_2$ will be along negative *x*-axis. For this current is anticlockwise as seen from the right side of the loop on the *x*-axis.
