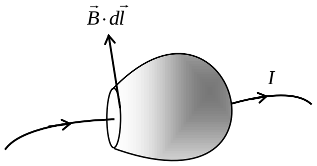
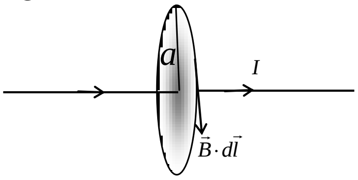
4. Ampere's Circuital Rule

1.State and prove Ampere's Circuital Rule.

▶ In an open surface with a boundary, The line integral of $\vec{B}$ along the boundary of an open surface of any shape is equal to μ_0 times the total current crossing the surface.





Mathematically, $\oint \vec{B}.d\vec{l} = \mu_0 i$.

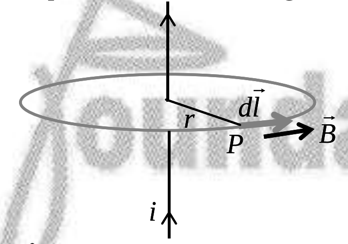
Proof: A circular amperean loop of radius *a* is drawn about a current carrying straight conductor having conductor as the axis. Magnetic field vector $\vec{B}$ will be tangential at every point of the loop. $\therefore \vec{B}\cdot d\vec{l} = Bdl = \frac{\mu_0 i dl}{2\pi a}$.[$\because$ $\vec{B} = \frac{\mu_0 i}{2\pi a}$].

Integrating for the whole loop, $\oint \vec{B}.d\vec{l} = \left(\frac{\mu_0 i}{2\pi a}\right)\oint dl = \left(\frac{\mu_0 i}{2\pi a}\right)2\pi a$.

$\Rightarrow \boxed{\oint B.d\vec{l} = \mu_0 i}$. This is *Ampere's circuital rule.*

2.Applying Ampere's Circuital rule, find the field due to a straight long conductor.

▶ Let there be a long straight conductor carrying current *i*. Let field is to be determined at a point *P* distant '*r*' from the conductor.
An amperean loop is drawn passing through *P*, about the conductor, having the conductor as its axis. By symmetry, at every point of this loop field *B* has same magnitude and will be directed tangentially.



$\therefore \oint \vec{B}\cdot d\vec{l} = \oint Bdl = B.2\pi r$ (Because $\oint dl = 2\pi r$).

By Ampere's rule, $\oint \vec{B}.d\vec{l} = \mu_0 i .\Rightarrow B.2\pi r = \mu_0 i$.$\therefore$ $\boxed{B = \frac{\mu_0 i}{2\pi r}}$.

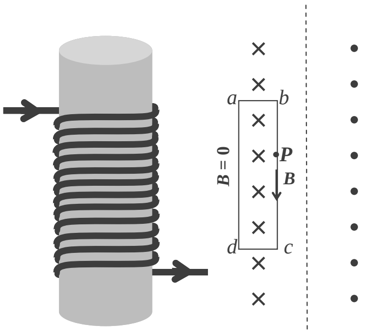
3.The magnetic field due to a long straight wire has been derived in terms of μ_0, *i* and *r*. Express this in terms of ε_0, *c*, *i* and *r*.

▶The magnetic field due to a long straight wire $B = \frac{\mu_0 i}{2\pi r}$.

By electromagnetic theory, $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$. Therefore $\mu_0 = \frac{1}{c^2 \varepsilon_0}$. $\Rightarrow B = \frac{i}{2c^2 \varepsilon_0 \pi r}$.

4.Field inside a solenoid: Prove that field is *uniform* inside the solenoid. (b) Find the field inside a toroid. Prove that field is non *uniform* inside the toroid.

▶ Let there be a long solenoid having *n* turns per unit length and radius *a*. In a longitudinal cross section, currents come out of the section from right row and goes into the page at the left row. If *i* current flows through the wire, each current has value *i*. Let there be a point P inside the solenoid. Drew an *amperean loop abcda* through point *P* in the plane of cross section of the solenoid including left row. Due to symmetry at every point on *bc* field will have the same magnitude.
Let it be *B*.



Outside the solenoid $B = 0$. $\therefore \oint \vec{B}.d\vec{l} = B[(ab)\cos 90^o + B(bc)\cos 0^o + (cd)\cos 90^o + 0] = B(bc)$.

By Ampere's circuital rule $\oint \vec{B}\cdot d\vec{l} = \mu\ i_{total}$.

The total current $i_{total} = n(bc)i$ [enclosed by the amperean loop].

Therefore, $B(bc) = \mu_0 n(bc)i$. $\Rightarrow$ $\boxed{B = \mu_0 ni}$.

Since the expression is independent of the position of the point *P the field is uniform inside the solenoid*.

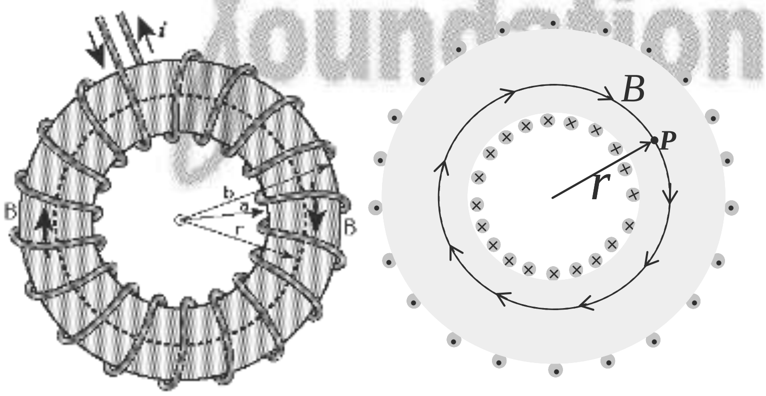
5.Field inside a toroid:

▶ Let there be a point *P* inside a toroid of total number of turns = *N*. Distance of point *P* from the center of the toroid is *r*. Drawn a circular amperean loop having centre same as that of the toroid through point *P*. At every point on the loop field *B* is same and is directed tangentially. $\therefore$ $\oint \vec{B}\cdot d\vec{l} = B\oint dl = B.2\pi r$, where *r* = radius of *amperean loop*.

If current *i* flows through the toroid then total current enclosed by the *amperean loop* = *Ni*.

By Ampere's circuital rule, $\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{total} \Rightarrow B.2\pi r = \mu_0 N i$. Therefore, $\boxed{B = \dfrac{\mu_0 iN}{2\pi r}}$

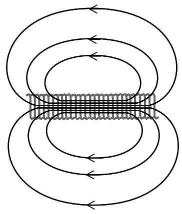
Since the field depends upon the position of the point *P* hence it is *not uniform* within the toroid.



Toroid

? Explain why field outside a long solenoid is taken to be zero.

Ans: The concentric circular magnetic field lines of the turns of a solenoid are packed inside the solenoid forming almost straight field lines parallel to the axis.



Outside the solenoid the same number of field lines are very much rarefied and spread over a very large space up to infinity. Larger the length of the solenoid rarer is the field outside. Hence outside a long solenoid the magnetic field is taken to be zero.

? A current is setup in a long copper pipe. Is there a magnetic field : (a) outside, (b) inside the pipe?
Ans: Using Ampere's rule we, find that over any Amperean loop around the pipe the line integral of field B can't be zero, hence there is field outside the pipe. But there is no current inside the pipe hence line integral of B over any Amperean loop is zero, therefore B is zero inside the pipe.

? In Ampere's law $\oint \vec{B}.d\vec{l} = \mu_0 i$, the current outside the curve is not included on the right hand side. Does it mean that the magnetic field B calculated by using Ampere's law, gives the contribution of only the currents crossing the area bounded by the curve?
Ans: No, all currents contribute to the field, e.g., field in a solenoid, field inside a current carrying wire etc. But Ampere's theorem is applied for calculating field in the symmetric cases, where all contributing currents are passing through the surface bounded by the loop.

? The magnetic field inside a tightly wound, long solenoid is $B = \mu_0 ni$. It suggests that the field does not depend on the total length of the solenoid, and hence if we add more loops at the ends of a solenoid the field should not increase. Explain qualitatively why the extra- added loops do not have a considerable effect on the field inside the solenoid.
Ans: Field due to a loop at a distance x from the center of the loop when radius of the loop $a << x$, is $B = \frac{\mu_0 i a^2}{2x^3}$. Therefore at a large value of x the field almost vanishes. A loop located at merely a distance $5a$ contributes only less than 1% of the total field $\mu_0 ni$.

? In order to have a current in a long wire, it should be connected to a battery or some such device. Can we obtain the magnetic field due to a straight, long wire by using Ampere's law without mentioning this other part of the circuit?
Ans: Yes we can, by neglecting the other part of the circuit and considering it to be at infinity. But Ampere's law requires a linkage between the path of the current and the path of the line integral of the B vector.

? Quite often, connecting wires carrying currents in opposite directions are twisted together in using electrical appliances. Explain how it avoids unwanted magnetic fields.
Ans: The twisting of wires gives net current zero enclosed by any Amperean loop round the twisted wire hence net zero magnetic field. In other words, physically the equal and opposite currents produce equal and opposite field which cancel each other.

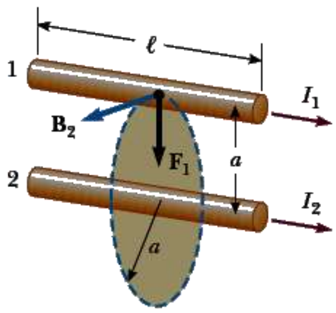
6.Force per unit length between two parallel conductors:

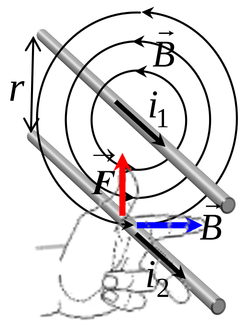
▶ Let there be two parallel straight conductors placed in vacuum at a distance r carrying current i_1 and i_2 in same direction. Magnetic field at any position on the conductor carrying current i_2, due to of current i_1 is $B = \frac{\mu_0 i_1}{2\pi r}$, perpendicular to the length of conductor.

Therefore force on an element of current i_2 is $dF = (\frac{\mu_0 i_1}{2\pi r}) i_2 dl$. Force per unit length $\frac{dF}{dl} = \frac{\mu_0 i_1 i_2}{2\pi r}$.

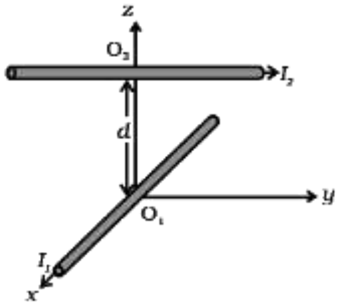
By *left hand rule* the direction is towards i_1.

By Newton's third law the same force is on the current i_1. Hence the force is attractive. If direction of one of the currents is reversed then force becomes repulsive.





⁇ Two long wires carrying current I_1 and I_2 are arranged as shown in the figure. The one carrying current I_1 is along is the x-axis. The other carrying current I_2 is along a line parallel to the y-axis given by $x = 0$ and $z = d$. Find the force exerted at O_2 because of the wire along the x-axis.

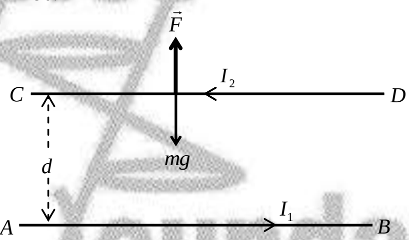


Ans: At O_2, the magnetic field due to I_1 is along the y-axis. The second wire is along the y-axis and hence the force is zero.

⁇ Do magnetic forces obey Newton's third law? Verify for two current elements $id\vec{l}_1 = idl\hat{i}$ located at the origin and $id\vec{l}_2 = idl\hat{j}$ located at (0, R, 0). Both carry current I.

Ans: No. Force due to $id\vec{l}_2$ on $id\vec{l}_1$ is zero. Force due to $id\vec{l}_1$ on $id\vec{l}_2$ is non-zero.

⁇ A wire AB is carrying a current of 12A and is lying on the table. Another wire CD, carrying a current of 5A, is arranged just above AB at a height of 1mm. What should be the weight, per unit length of this wire so that CD remains suspended at its position? Indicate the direction of the current in CD and the nature of force between the two wires.



Ans: I_1 = 12 A, I_2 = 5A, d = 1mm.
The current in CD must be opposite to that in AB so that it acts upwardly to balance the weight of the wire CD.

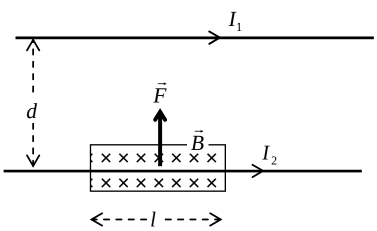
Therefore force of repulsion between the wires acting on CD upwardly is $F = \dfrac{\mu_0 I_1 I_2}{2\pi d}$ on unit length.

If weight of the unit length of the wire CD is w then for balance, $w = \dfrac{\mu_0 I_1 I_2}{2\pi d}$.

Placing numerical values, $w = (2\times10^{-7}\times 12\times5)/(1\times10^{-3}) = 12$mg/m

⁇ A steady current (I_1) flows through a long straight wire. Another wire carrying steady current (I_2) in the same direction is kept close and parallel to the first wire. Show with the help of a diagram how the magnetic field due to the current I_1 exerts a magnetic force on the second wire. Write the expression for this force.

Ans:



Force acting on the wire carrying current I_2 over length l is $F = \dfrac{\mu_0 I_1 I_2 l}{2\pi d}$.