1. Magnetic Dipole

1.Magnet:

▶ A magnet is a kind of dipole that produces magnetic field and when placed in a magnetic field aligns itself along the magnetic field. Earth is a large magnet. A magnet when suspended freely in earth's magnetic field, aligns itself in North-South direction. The pole that points north, is called 'north pole' and that points south is called 'south pole' of the magnet.

2.Natural and artificial magnets:

▶ **Natural Magnets**: Naturally occurring iron ore magnetite (Fe_3O_4), also called loadstone or rock iron, is natural magnet because they have capability of attracting iron bits. Greeks knew about the property of magnet (lodestone) that it attracts iron as early as 600B.C. The directional property of the magnet that it points north-south when suspended freely, was also known and used or ship navigation by the Chinese, as early as 400B.C.

Artificial Magnets: A hard magnetic material (such as hard iron) is magnetized by strong external magnetic field, or by rubbing loadstone (natural magnet) with hard it. This magnetized iron is called *artificial magnet*. Artificial magnets are made as bar magnets, horse-shoe magnets, magnetic needle etc.

3.Electromagnet:

▶When a soft iron core is placed in a current carrying solenoid, the iron core is magnetized and on breaking the current, the magnetization vanishes. Such a magnet is called electromagnet. It is used in electric bell, Rumkorff's induction coil, magnetic crane etc.

4.What are magnetic field lines:

▶ To understand the behavior of magnetic field, the notion of magnetic field lines, analogous to electric field lines, is used. It is defined as: *Magnetic field lines are the curves along which a magnetic dipole aligns itself in a magnetic field.*

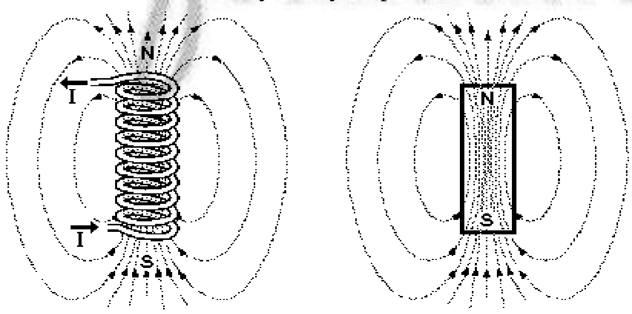
The magnetic field lines have the following characteristics:

(i) Magnetic field lines are closed continuous curves[1] like closed loops, i.e., they have no starting or ending points.

(ii) Tangent to the magnetic field lines gives the direction of the magnetic field $\vec{B}$ at that point (*remember, not the direction of* ***magnetic force***).

(iii) Number of lines per unit cross sectional area is proportional to the magnitude of $\vec{B}$.

(iv) No two field lines intersect each other, they always repel each other.

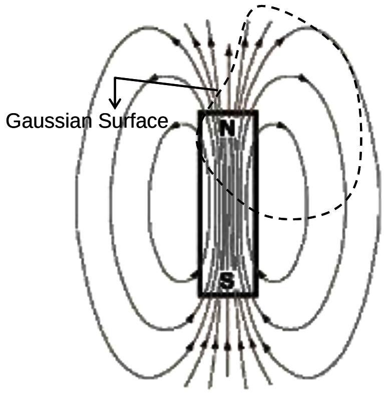


5.Magnetic flux & Gauss's theorem in magnetostatics:

▶ **Magnetic flux:** Product of the magnetic field and an area of a surface perpendicular to the magnetic field is called magnetic flux. If magnetic field crosses over an extended surface plane or curved and at any point over elementary plane area *ds* its value is $\vec{B}$ then the magnetic field perpendicular to the area will be $B\cos\theta$, where θ is the angle between the normal to the plane area ds ($\hat{n}$) and $\vec{B}$. Therefore by definition the magnetic flux across *ds* will be $d\phi_B = Bds\cos\theta$, or $d\phi_B = \vec{B}\cdot d\vec{s}$, where $d\vec{s} = ds\hat{n}$ is the area vector at the given point on the surface. The total flux over the entire surface will be the integration of the $d\phi$ over the entire surface, i.e., $\phi = \int \vec{B}\cdot d\vec{s}$.

In other words, the surface integral of $\vec{B}$ over an area is the total flux through the surface. It is also directly proportional to the number of field lines passing the area.

[1] Contrary to electric field lines which are open curves.



Gauss's theorem:

Statement: Total magnetic flux of $\vec{B}$ over a closed surface is always equal to zero. $\Rightarrow \oint \vec{B}\cdot d\vec{s} = 0$.

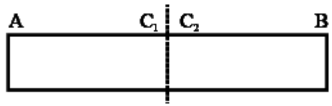
Proof: Since magnetic lines of forces are continuous, hence across any closed surface total field lines entering in to the closed surface equals total lines emerging out of the closed surface.
Hence net flux = 0. $\therefore \oint \vec{B}.d\vec{s} = 0$.

❓ What is the significance of Gauss's law in magnetism?
Ans: Gauss's law of magnetism acknowledges about non-existence of magnetic monopoles.
❓ If the magnetic monopole were to exist, how would the Gauss's law of magnetism get modified?
Ans: If monopoles were existed the Gauss's theorem would have form $\oint \vec{B}.d\vec{s} = \mu_o q_m$ where q_m is the strength of the magnetic charge or monopole.

6.Magnetic dipole moment of a bar magnet:

▶ A bar having two magnetic poles of equal pole strengths *N* and *S* is called a *magnetic dipole* or a *bar magnet*. The product of magnetic pole strength (q_m) and magnetic length 2*l* is called *magnetic dipole moment* of the bar magnet. Therefore, $\boxed{\vec{P}_m = 2q_m\vec{l}}$.
Its direction is from *S* to *N*. North pole is called +ve magnetic charge and south pole, –ve magnetic charge.

❓ What will be (i) Pole strength (ii) Magnetic moment of each of new piece of bar magnet if the magnet is cut into two equal pieces: (a) Normal to its length? (b) Along its length?
Ans: (a) (i) no change, (ii) becomes half, (b) (i) becomes half, (ii) becomes half.
❓ A (hypothetical) bar magnet (AB) is cut into two equal parts. One part is now kept over the other, so that pole C_2 is above C_1. If M is the magnetic moment of the original magnet, what would be the magnetic moment of the combination so formed?



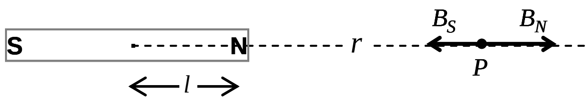
Ans: Zero.
❓ Must every magnetic field configuration have a north pole and a south pole? What about the field due to a toroid?
Ans: No. Magnetic poles are present in the field which has a dipole moment. Toroid has no dipole moment so has no pole. [The magnetic lines are closed curves and have no emanation and termination point.]

7.Magnetic field strength at a point:

▶ **(i) On the axis of the magnet**:
Let there be a point *P* distant *x* from the centre of the dipole. By Coulomb's law in magnetostatics, field at *P* due to north pole is $B_N = \dfrac{\mu_0}{4\pi}\dfrac{q_m}{(r-l)^2}$. Field due to south pole is $B_S = \dfrac{\mu_0}{4\pi}\dfrac{q_m}{(r+l)^2}$ (opposite to B_N).

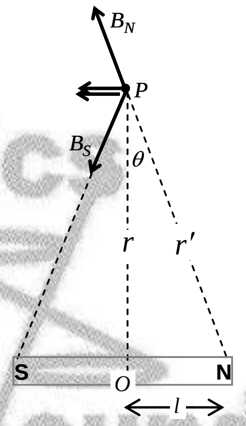
Therefore net field at P is $B = \frac{\pi_0}{4\pi} q_m \left(\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2} \right)$.

$\Rightarrow B = \frac{\mu_0 q_m}{4\pi} \frac{4rl}{\left(r^2 - l^2\right)^2}$. $\Rightarrow \vec{B} = \frac{\mu_0}{4\pi} \frac{\left(2\vec{P}_m\right)r}{\left(r^2 - l^2\right)^2}$. If $l << r$ then $\vec{B} = \frac{\mu_0}{4\pi} \frac{2\vec{P}_m}{r^3}$



(ii) On the $\perp^{er}$ bisector of a magnetic dipole:
Let there be a point P on the $\perp^{er}$ bisector of a magnetic dipole distant r from the centre O. Let r' is the distance of P from the poles. Field due to N-pole (along $\overrightarrow{NP}$) $B_N = (\mu_0 q_m / 4\pi r'^2)$.

Field due to S -pole (along $\overrightarrow{PS}$) is $B_S = (\mu_0 qm / 4\pi r'^2)$.

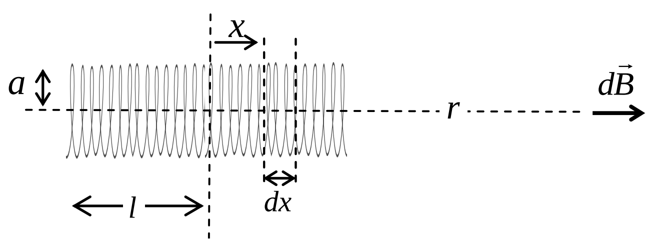


Since, $|B_N| = |B_S|$, hence components of B_N and B_S along bisector conceals each other. Components $\perp^{er}$ to the bisector add up. Therefore net field is anti parallel to the dipole moment of the magnet. $B = 2\frac{\mu_0 q_m}{4\pi r^2} \sin\theta$. $\Rightarrow B = \frac{\mu_0 2 q_m l}{4\pi r^3}$. For $l << r'$, $\Rightarrow r \approx r'$, hence $\vec{B} = -\frac{\mu_0 \vec{P}_m}{4\pi r^3}$.

8. Find the magnetic field on the axis of a short solenoid at a distance *r* from the center of the solenoid. Hence prove that a solenoid is equivalent to a magnetic dipole.

▶ Supposed there is a solenoid of length $2l$ and radius a and number of turns per unit length n. It carries current i. At a point P on the axis of the solenoid at a distance r from the centre of the solenoid the field is to be calculated where $r >> l$. Supposed at distance x from the center there is an elementary portion of the solenoid of width dx.

The field at P due to this element will be $dB = \frac{\mu_0 a^2 i n dx}{2[(r-x)^2 + a^2]^{3/2}}$.



Since $r >> l$, hence $r >> x$ also, therefore neglecting x from the denominator we have, $dB = \frac{\mu_0 a^2 i n dx}{2(r^2 + a^2)^{3/2}}$.

The total field at P is calculated by integrating dB from $x = -l$ to $x = +l$.

That is, $B = \int_{-l}^{l} \frac{\mu_0 a^2 indx}{2(r^2 + a^2)^{3/2}}$. $\Rightarrow B = \frac{\mu_0 a^2 in}{2(r^2 + a^2)^{3/2}}[x]_{-l}^{l}$. $\Rightarrow B = \frac{\mu_0 a^2 inl}{(r^2 + a^2)^{3/2}}$.

Multiplying numerator and denominator by 4π we have $B = \frac{\mu_0 2[(\pi a^2 i)(n2l)]}{4\pi(r^2 + a^2)^{3/2}}$.

In the numerator, $(\pi a^2 i)(n2l)$ is the sum of magnetic moments of the all $n2l$ loops of the solenoid, therefore this is the total magnetic moment of the short solenoid, that is $P_m = (\pi a^2)(n2l)$.

Hence, $B = \frac{\mu_0 2P_m}{4\pi(r^2 + a^2)^{3/2}}$. If $a << r$ then, $B = \frac{\mu_0 2P_m}{4\pi r^3}$.

Therefore field due to the short solenoid at a distance r from the center on its axis is:

$\vec{B} = \frac{2\mu_0 \vec{P}_m}{4\pi r^3}$, where, $P_m = (\pi a^2)(n2l)i$.

This is analogous to the field due to a magnetic dipole which is given as $\vec{B} = \frac{2\mu_0 \vec{P}_m}{4\pi r^3}$.

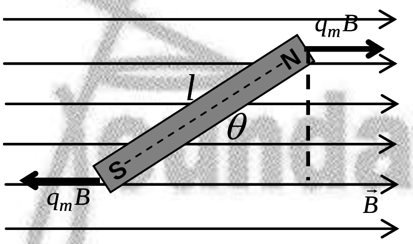
Therefore a short solenoid is equivalent to a magnetic dipole.

9. Calculate torque on a magnetic dipole placed in a uniform field *B*.

▶Let the axis, hence dipole moment vector of a bar magnet $\vec{P}_m$ makes angle θ with the direction of $\vec{B}$. Magnetic length = l and pole strength = q_m.

The equal and opposite magnetic forces on the poles $F = q_m B$ form a couple. The lines of action of the forces are separated by a distance $l\sin\theta$.

Therefore couple has moment $\tau = q_m Bl\sin\theta$. $\Rightarrow \tau = P_m B\sin\theta$. $\Rightarrow \vec{\tau} = \vec{P}_m \times \vec{B}$.



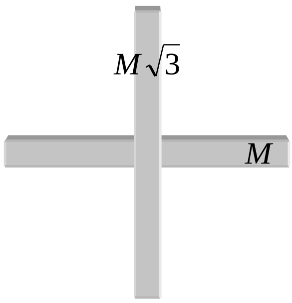
‽ A short bar magnet has magnetic moment of 50 A m². Calculate the magnetic field intensity at a distance of 0.2 m from its centre on (1) its axial line (2) its equatorial line.

Ans:(1) 1.25 mT (2) 0.625 mT

‽ Calculate the torque acting on a magnet of length 20 cm and pole strength 2×10^{-5} Am, placed in the earth's magnetic field of flux density 2×10^{-5} T, when (a) magnet is parallel to the field (b) magnet is perpendicular to the field.

Ans: (a) 0, (b) 8×10^{-11} N-m

‽ Two magnets of magnetic dipole moments M and M√3 are joined to form a cross. The combination is suspended in a uniform magnetic field B. The magnetic moment M now makes an angle θ with the field direction. Find the angle θ.



Ans: 60°

10. Potential Energy and Work done in rotating a magnetic dipole in a uniform magnetic field:

▶ (i) Let there be a magnetic dipole of dipole moment P_m placed in a uniform field B and $\vec{P}_m$ makes angle θ with $\vec{B}$. Instantaneous torque $\tau = P_m B \sin\theta$.

The work done in deflecting the dipole by angle $d\theta$ against this torque $d\omega = P_m B \sin\theta\, d\theta$. Integrating from initial angle θ_1 to final angle θ_2 we have, $W = \int_{\theta_1}^{\theta_2} P_m B \sin\theta\, d\theta$. $\Rightarrow W = P_m B(\cos\theta_1 - \cos\theta_2)$.

(ii) Since work done on the dipole in rotating it equal to the change in its potential energy therefore, $W = U_2 - U_1 = P_m B(\cos\theta_1 - \cos\theta_2)$.

If initially the dipole was at $\theta_1 = 90°$ where $U_1 = 0$ and final position is $\theta_2 = \theta$, where $U_2 = U$, then $U = -P_m B\cos\theta$ or $U = -\vec{P}_m \cdot \vec{B}$.

11. Find the expression for the time period of small oscillation of a magnetic dipole placed in a uniform field.

▶If a magnetic dipole is free to rotate about an axis possessing through its centre, it orients itself to zero torque position i.e. $\vec{P}_m$ becomes parallel to $\vec{B}$.

If the dipole is deflected slightly and left, it oscillates about the axis. Supposed at a position the magnetic moment of the dipole makes an angle θ with $\vec{B}$.

$\therefore \tau = P_m B\sin\theta$. If θ is very much small then $\sin\theta = \theta$ and $\tau = P_m B\, \theta$.

If I = moment of inertia of the magnet and α is its instantaneous acceleration then, $I\alpha = -P_m B\theta$ (θ and α are in opposite directions). $\Rightarrow \alpha = -(P_m B / I)\theta$. $\Rightarrow \alpha \propto -\theta$.

This suggests that motion of the magnet is SHM and the period is $T = 2\pi\sqrt{(I / P_m B)}$.

12. List the electrostatic analogue of the magnetic quantities.

▶ The following table shows the electrostatic analogue of the magnetic quantities:

	Electric	**Magnetic**
Constant for vacuum	$\frac{1}{\varepsilon_0}$	μ_0
Dipole moment	$\vec{p}$	$\vec{m}$ **or** $\vec{p}_m$
Equatorial field for a short dipole	$-\frac{\vec{p}}{4\pi\varepsilon_0 r^3}$	$-\frac{\mu_0 \vec{p}_m}{4\pi r^3}$
Axial field for a short dipole	$-\frac{2\vec{p}}{4\pi\varepsilon_0 r^3}$	$-\frac{2\mu_0 \vec{p}_m}{4\pi r^3}$
Torque in external field	$\vec{p} \times \vec{E}$	$\vec{p}_m \times \vec{B}$
Potential energy	$-\vec{p} \cdot \vec{E}$	$-\vec{p}_m \cdot \vec{B}$